ChE330 Fall 2021 Final Exam KEY

1) ⓐ $v_\infty = 0$ heat transfer can only occur by conduction. Consider a sphere with Radius R transferring heat to surroundings when $v_\infty = 0$ we can write (for $q|_{r=R}$)

$$-k\frac{\partial T}{\partial r}\bigg|_{r=R} = h\left(T|_{r=R} - T_\infty\right)$$

T_∞ @ $r=\infty$

Recall that $Nu_D = \frac{hD}{k}$

we can write a steady state differential balance for heat conduction in "surroundings" and solve for "h" as follows:

$0 = k\nabla^2 T$ or since $T = T(r)$ in spherical coordinates:

$$0 = \frac{k}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) \quad \text{with B.C.'s as: } \begin{bmatrix} T(r=R) = T_s \\ T(r=\infty) = T_\infty \end{bmatrix}$$

$$\Downarrow$$

$$r^2\frac{dT}{dr} = C_1$$

$$\frac{dT}{dr} = \frac{C_1}{r^2}$$

$$T = C_2 - \frac{C_1}{r}$$

apply B.C.'s

$$T_\infty = C_2$$

$$T_s = T_\infty - \frac{C_1}{R}$$

$$\Downarrow$$

$$-R(T_s - T_\infty) = C_1$$

thus

$$\boxed{T = T_\infty + \frac{R(T_s - T_\infty)}{r}}$$

then we can solve for $q|_{r=R}$

$$q|_{r=R} = -k \frac{dT}{dr}\Big|_{r=R}$$

$$= \frac{+kR(T_S - T_\infty)}{R^2}$$

$$= \frac{k(T_S - T_\infty)}{R}$$

we can equate this to $h(T_S - T_\infty)$

$$\frac{k\cancel{(T_S - T_\infty)}}{R} = h\cancel{(T_S - T_\infty)}$$

thus $\boxed{h = \frac{k}{R}}$

Plug this into $Nu = \frac{hD}{k}$

$$\boxed{Nu = \frac{\frac{k}{R} \cdot D}{k} = \frac{\frac{\cancel{k}}{\cancel{R}} \cdot 2\cancel{R}}{\cancel{k}} = 2}$$

2) in the glass continuum $T = T(z)$
Steady state, no heat production.
thus $0 = k\nabla^2 T$ or $0 = \nabla^2 T$ in 1D then

$0 = \frac{d^2T}{dz^2}$ with Boundary Conditions! 3

@ $z=0$

$$q|_{z=0} = h_0 (T|_{z=0} - T_0)$$

OR

$$-k_g \frac{dT}{dz} = h_0 (T|_{z=0} - T_0)$$ 3

@ $z=L$ the heat produced will be both conducted into glass and convected into air thus we can write

$$S_A = |q_{cond}| + |q_{conv.}|$$

OR

$$S_A = k_g \frac{dT}{dz} + h_i (T|_{z=L} - T_i)$$

Solve differential eq. $\frac{dT}{dz} = C_1$

$$T = C_1 z + C_2$$

apply B.C. @ $z=0$

$$-k_g C_1 = h_0 (C_2 - T_0)$$

apply B.C. @ $z=L$

$$S_A = k_g C_1 + h_i \{(C_1 L + C_2) - T_i\}$$

From the eq. @ $z=0$

$$-k_g C_1 = h_o C_2 - h_o T_o \Rightarrow C_2 = \frac{h_o T_o - k_g C_1}{h_o}$$

OR

$$C_2 = T_o - \frac{k_g}{h_o} C_1$$

then plug into eq. for $z=L$

$$S_A = k_g C_1 + h_i \left[C_1 L + \frac{h_o T_o - k_g C_1}{h_o} - T_i \right]$$

expand

$$S_A = k_g C_1 + h_i L C_1 + h_i T_o - \frac{k_g h_i}{h_o} C_1 - h_i T_i$$

Rearrange!

$$S_A + h_i (T_i - T_o) = C_1 \left\{ k_g + h_i L - \frac{h_i k_g}{h_o} \right\}$$

thus

$$\boxed{C_1 = \frac{S_A + h_i (T_i - T_o)}{k_g + h_i L - \frac{h_i k_g}{h_o}}}$$

and

$$\boxed{C_2 = T_o - \frac{k_g}{h_o} \left[\frac{S_A + h_i (T_i - T_o)}{k_g + h_i L - \frac{h_i k_g}{h_o}} \right]}$$

4

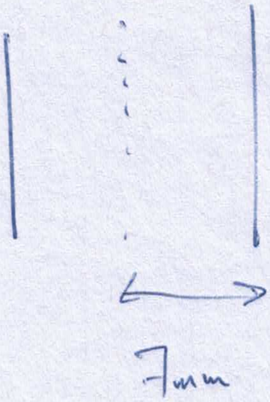
Finally!

$$\boxed{T(z) = C_1 z + C_2}$$

with C_1 & C_2 as written above!

3)

12



$k = 6\ W m^{-1} K^{-1}$

$c_p = 850\ J\ kg^{-1} K^{-1}$

$\rho = 5200\ kg\ m^{-3}$

$h = 500\ W m^{-1} K^{-1}$

$T_\infty = 95°C$

$T_i = 18°C$

$T(t=?) = 44°C$

1st calculate $Bi_L = \frac{hL}{k} = \frac{500 \cdot 0.007}{6} = 0.58333$

then from tables: $\lambda_1 = 0.69 \quad A_1 = 1.08$

at the "center" $\theta = \frac{T_0 - T_\infty}{T_i - T_\infty} = A_1 e^{-\lambda_1^2 \tau}$ $\quad \tau > 0.2$

$$\frac{44-95}{18-95} = \frac{-51}{-77} = 0.662 = 1.08 \exp(-0.69^2 \tau)$$

$$+0.4889 = +0.69^2 \tau$$

$$\tau = 1.03 \quad (>0.2!) \checkmark$$

$\tau = \frac{\alpha t}{L^2} \qquad \alpha = \frac{k}{\rho c_p} = \frac{6}{5200 \cdot 850} = 1.357 \times 10^{-6}$

$$t = \frac{\tau L^2}{\alpha} = \frac{1.03 \cdot (0.007)^2}{1.357 \times 10^{-6}} = 37.2\ \text{sec}$$

4 or 4) mass exchanger

in analogy to heat transfer.

$$N = \sqrt{\frac{4 k_m L^2}{D \mathcal{D}}}$$

where k_m is some mass transfer coefficient and $\mathcal{D}$ is some diffusion coefficient.

the units of $\frac{m^2}{s}$ and N is adimensional

then

$$\frac{\left[\frac{m}{s}\right]\left[m^2\right]}{[m]\left[\frac{m^2}{s}\right]}$$ k_m must have units $\frac{m}{s}$

then

$$N = \sqrt{\frac{4 k_{c,L} L^2}{D \mathcal{D}_{CO_2, cal.carb.}}}$$ 6

where

$k_{c,L}$ is the mass transfer coefficient Based on the liquid phase with respect to concentration

and $\mathcal{D}_{CO_2, cal.carb}$ is the diffusion coefficient for CO_2 in the calcium carbonate

5) Drying Methane

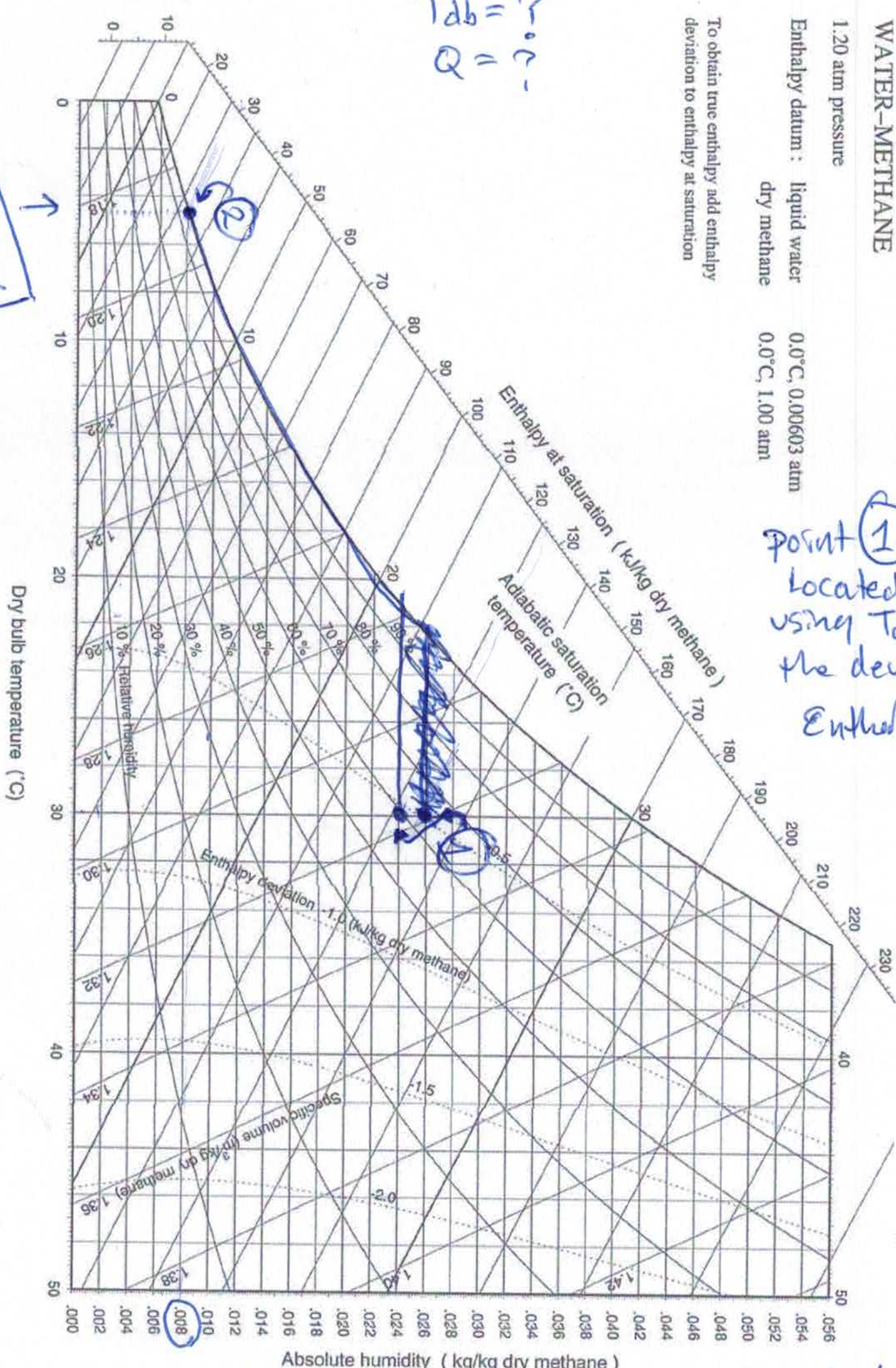
Cooler

① $p = 1.2\,atm$ methane $T_{db} = 30°C$ dew point = 24.2°C → [Cooler] → Q

② 1.09 kg/s, 0.008 kg H_2O / kg dry CH_4 → Rxn

$T_{db} = ?$

$Q = ?$



$\boxed{4.6°C}$

Point ① can be located on the chart using T_{db} and considering the dew point.

Enthalpy ≈ 125 $\frac{kJ}{kg\,dry\,methane}$

upon exiting the cooler the CH_4 will be saturated thus point ② must be as indicated. where $\boxed{T_{db} = 4.6°C}$ enthalpy = 30 $\frac{kJ}{kg}$

$$\left[125\frac{kJ}{kg} - 30\frac{kJ}{kg}\right]1.09\frac{kg}{s} = 103.6\frac{kJ}{s} = \boxed{104\ kW}$$